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Tracing the Evolution of Current Automatic Proving Technologies

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Given its formal, logical, and spatial properties, geometry is well suited to teaching environments that include dynamic geometry systems (DGSs), geometry automated theorem provers (GATPs), and repositories of geometric problems. These tools enable students to explore existing knowledge in addition to creating new constructions and testing new conjectures. With the help of a DGS, students can visualise geometric objects and link the formal, axiomatic nature of geometry (e.g., Euclidean geometry) with its standard models and corresponding illustrations (e.g., the Cartesian model). With the help of GATPs, students can check the soundness of a construction (e.g., if two given lines are parallel) and also create formal proofs of geometric conjectures. Supported by repositories of geometric knowledge, these tools provide teachers and students with a framework and a large set of geometric constructions and conjectures for doing experiments.

The evolution of current automatic proving technologies is traced [14]. How these technologies are beginning to be used by geometry practitioners in general to validate geometric conjectures and generate proofs with natural language and visual rendering, and foresee their evolution and applicability in an educational setting. Following Gila Hanna's [5, p.8] argument that "the best proof is one that also helps understand the meaning of the theorem being proved: to see not only that it is true, but also why it is true," and the large number of articles on proof and proving in mathematics education from the ICMI Study 19 Conference [12, 13], the focus must be on practices of verification, explanation, and discovery in the teaching and learning of geometry.

In the classroom, the fundamental question a proof must address is "why?" In this context, then, it is only natural to view proofs first and foremost as explanations and, as a consequence, to give more value to those that provide a better explanation. Dynamic geometry systems encourage both exploration and proof because they make it so easy to pose and test conjectures. The feature that preserves manipulations allows students to explore "visual proofs" of geometric conjectures. Such a powerful feature gives them strong evidence that a theorem is true and reinforces the value of exploration by giving them confidence on the truthfulness of a given geometric property.

The challenge facing classroom teachers is how to use the excitement and enjoyment of exploration to motivate students while also explaining that visual exploration is not a proof.

Visual exploration is a useful aid, but is still only the exploration of a finite number of cases. One reason for giving students a formal proof is that exploration does not reflect the need for rigour in mathematics. Indeed, mathematicians aspire to a degree of certainty that can only be achieved by a proof. A second reason is that students should come to understand the first reason. As most mathematics educators would agree, students need to be taught that exploration, useful as it may be in formulating and testing conjectures, does not constitute a proof [5, 6]. A proof is a means of obtaining certainty about the validity of a conjecture (proof as a validation tool) and a strategy to further understand a formulated conjecture (proof as an instrument of understanding).

Geometry automated theorem provers open the possibility of formally validating properties of geometric constructions. For example: *Cinderella*[*] [16] has a randomised theorem checker; *Java Geometry Expert* (*JGEX*)[†] [20], *Geometry Constructions LaTeX Converter* (*GCLC*)[‡] [8] and *GeoGebra*[§] (version 5) [7] incorporate a number of automated theorem provers that provide a formal answer to a given validation question [2, 10].

Automated deduction techniques also enable students to explore new knowledge and discover new results and theorems [19] (e.g., the algebraic formula of a loci [1, 15]). An important addition to any learning environment would be a GATP with the ability to produce human readable formal proofs with, eventually, visual counterparts [3, 4, 9, 11, 17, 18].

Keywords
Dynamic Geometry, Computer Algebra, Automated Deduction, Computational Tools in Education

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[*] `https://cinderella.de`
[†] `http://www.cs.wichita.edu/~ye/`
[‡] `http://poincare.matf.bg.ac.rs/~janicic/gclc/`
[§] `https://www.geogebra.org/`

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