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Automated Plane Geometry in Wolfram Mathematica

Daniel McDonald[1]***, Peter Barendse***[1,2] [dmcdonald@wolfram.com]

[1] Wolfram Research Institute, Champaign, Illinois, USA
[2] Department of Materials Science and Engineering, Massachusetts Institute of Technology, Boston, Massachusetts, USA

We discuss new tools in the Wolfram Language (the language of the computing system Mathematica) for automatically drawing as well as making conjectures and proving theorems about symbolically described, coordinate-free scenes in plane geometry. These new functions include `GeometricScene`, `RandomInstance`, `FindGeometricConjectures`, and `FindGeometricProof`, which together support the following workflow.

1. `GeometricScene` allows a user to describe a coordinate-free scene in plane geometry.
2. `RandomInstance` draws a randomized instance of the scene.
3. `FindGeometricConjectures` makes conjectures about the scene.
4. `FindGeometricProof` gives human-readable proofs of theorems that hold given the hypotheses of the scene.

`GeometricScene`, `RandomInstance`, and `FindGeometricConjectures` are currently available in Mathematica Version 12, while `FindGeometricProof` will be introduced in a future version. This talk will address the following aspects of these functions.

1. A `GeometricScene` object contains lists of symbolic point coordinates and scalar parameters, which may or not be assigned numerical values, followed by a list of hypotheses describing a scene involving those points and parameters, with a final optional list of potential conclusions drawn from the hyptheses. The contents of the hypotheses and conclusions must be written within the Wolfram Language framework to be simultaneously general enough to describe any given scene in planar geometry, specifically descriptive enough to allow succinct scene descriptions, and simple enough to be accessible to high school students.

2. `RandomInstance` adds coordinate and parameter values to a `GeometricScene` object by first generating and then nondeterministically solving a constrained optimization problem with those symbolic coordinates and parameters as variables. The `GeometricScene` object stores these values and formats itself as the corresponding graphic.

3. `FindGeometricConjectures` uses the coordinate and parameter values found by `RandomInstance` and stored in a `GeometricScene` object to search for interesting relations that hold in the given instance(s) of the scene.

4. `FindGeometricProof` returns logically sound, human-readable proofs using geometric, not algebraic, reasoning, with redundant or irrelevant steps excised.

`RandomInstance` is an example of a *geometric constraint solver*; for a general discussion of geometric constraint solving, see [2]. `FindGeometricProof` is an example of an *automated theorem prover*; for a general discussion of automated theorem proving in geometry, see [1].

Keywords
geometric constraint solver, automated theorem prover, plane geometry, Euclidean geometry, synthetic geometry

References
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