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One method of trisecting an angle and its interpretation for teaching purposes using a dynamic geometry and computer algebra system

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This contribution is focused on the use of a dynamic geometry and computer algebra system in mathematics education, namely in teaching at secondary schools and in the teaching of future mathematics teachers of lower and upper secondary schools. It presents the use of the software to interpret historical geometrical subject matter from the perspective of up to date mathematics, to create a dynamic model of the respective phenomenon and also to serve as a basis to create its physical model.

The contribution deals with a method of trisecting an angle [5] that was developed by J. R. Vaňaus, Czech mathematician, in his paper *Trisektorie* published in 1881 [7].

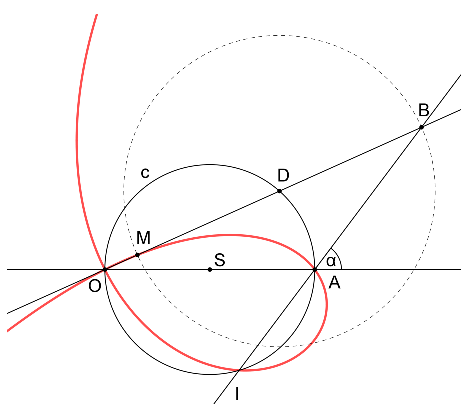

Figure 1: Vaňaus' trisectrix

Let us start the introduction of this method by presenting the example that Vaňaus assigned to readers of the Czech "Journal for doing mathematics and physics" in 1902: *Given a line segment AB. Circular arcs, both with the radius $|AB|$, are drawn around points A and B, passing through points B and A respectively and intersecting at point C. The task is to set* *points M and N at arcs AC and BC respectively so that the line segment MN is parallel to AB and the angle $\angle MAN$ is equal to a given acute angle.* [8] Three solutions to this problem, all leading to the trisection of an angle, sent by students of upper secondary school, were published in the last issue of the journal volume. In his comment to the solutions Vaňaus mentioned his 1881 paper in which he introduced a method of doing a trisection using the cubic curve shown in Fig. 1.

This cubic curve, currently known as the oblique strophoid [6, 4, 3], is presented by him as the locus of points M for B moving along the line l, a secant to the circle c, so that $|MD| = |DB|$, where D is the intersection of the line SB with c. He derives the equation of this curve and describes a simple way of using it to trisect an angle (the angle α in Fig. 1). In conclusion he mentiones his assembly of a simple mechanism to implement this trisection.

In this contribution we will show the use of the dynamic geometry and computer algebra features of GeoGebra software [2] to create a dynamic model of the respective geometric construction, to derive an equation of the curve and to design a virtual model of the mechanical linkage for the manual execution of the trisection. We will show that supported by the means of the automated theorem proving implemented into the dynamic geometry environment of GeoGebra [1] such tasks are at a corresponding level of complexity already feasible at secondary school.

Keywords
DGS, CAS, trisection, strophoid, mathematics education

References

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