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Rearrangement method for area of a circle: complex paths from historical roots to modern visual and dynamic models in discovery-based teaching approach

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The Internet is full of resources of all kinds (blogs, lesson plans, applets) aiming to introduce the formula for area of circle with 'discovery-based' methods. For instance, [1] suggests considering a circle of radius r as a cake which can be divided in a large number of equal slices (sectors). The slices could then be rearranged pointing alternately up and down to form a shape which looks like a 'rectangle' whose dimensions would be, on one side, radius, and on another side, close to the half of the circumference (which is known as being equal to $2\pi r$), so the area of the 'rectangle' and, therefore, the area of the circle would be $2\pi r^2/2 = \pi r^2$. The author adds that the closeness to 'rectangle' increases when the number of slices increases.

One resource suggested by the NCTM for Grade 7 students presents the same idea as a 'hands-on' activity of cutting (first in 8 sectors, then in 16 sectors) and rearranging the pieces in such a way that students would eventually 'see' a figure originally looking like a parallelogram which is getting closer to the 'rectangle' and then the formula for the circle is obtained from that of the area of the rectangle; see [2]. Using similar ideas, the LearnAlberta provides an interactive animation which allows to increase the number of sides (using a slider), so the rearrangement rapidly approaches the shape of a rectangle, see [3]. A GeoGebra applet created by Ooi Soo Huat, available at [4], allows for some more sophisticated exploration using several sliders to arrive at similar conjectures for the area of the circle.

Old methods of calculation of the area of a circle as applied to the teaching of infinitesimal procedures in the 20th century is an interesting case study within our ongoing project aiming at better understanding of historical roots of didactical approaches [5]. It was widely introduced in many mathematical treatises and textbooks produced since antiquity in East and West to became prominent in Western school textbooks in the second part of the 20th century. In its general form, the procedure can be summarized as follows: the circle is to be subdivided into a large number of identical sectors formed by the radii and the arcs of circumference between their ends. The area of the circle is approximated with the sum of the areas of the triangles having the radii as their long sides. This sum tends to the area of the circle when the number of the triangles grows indefinitely.

Historically, there existed various versions of this procedure slightly different from one another; they will be briefly discussed in our paper. In all the cases the inspected versions of this procedure were based on intuitive understanding of the concept of limit and were not accompanied by rigorous justifications. Similarly, the versions of this procedure found in modern school textbooks did not contain rigorous proofs; instead, they were appealing to the intuition of the learners helping them to 'discover' the formula, both, visually and dynamically.

In this our presentation we will briefly introduce the earliest specimens of this procedure, one found in the commentary of the Chinese mathematician Liu Hui (fl. AD 263) and the other in the manuscripts of Leonardo da Vinci (1452-1519). Then we will pass to the treatment of the area of circle found in West European school textbooks in the late 19th and early 20th century. After that we will investigate who, when, and under what circumstances injected in the school textbooks the method of calculation of area visibly similar to the methods of Liu Hui and Leonardo and discuss the historical background and hypothetical rationale of this didactical innovation. We will conclude the paper with a discussion of the current situation with the use of rearrangement method which can be found in many textbooks produced in a number of countries, as well as in online resources, aiming to a large variety of learners starting from Middle School Grades and towards modern college and undergraduate courses in calculus.

Keywords
Area of circle, Rearrangement formula, History and Modern Teaching.

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