Applications of Computer Algebra – ACA 2019
Montréal, Canada | July 16-20, 2019
École de technologie supérieure

The realization of a proof support system in a process of adaptation to the human perspective

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1 Context

Although the intelligent tutoring software QED-Tutrix is functional, its successful implementation into the context of a classroom requires an abundant supply of well thought out geometry problems. The goals of this software is to allow teachers to input their own problems in QED-Tutrix and to follow the student's thought process as much as possible while resolving the problem. To decrease the work involved in the complicated task of manually adding a new problem to the software, we developed an automated tool for the generation of mathematical proofs [1]. This automated tool has two main issues. First, the format in which problems are entered requires a reformulation of their typical statement to adapt them to the software's specifications. Second, the proofs obtained by this tool are often very detailed and rigorous due to the generation of every demonstration step, however sometimes obvious for both the teacher and student. Therefore, an improvement in this automated tool must be made for its use in a classroom context. Researchers in the Laboratoire Turing* work on two avenues with the goal of adapting the generated proofs: (1) the automated extraction of hypotheses and conclusions from problem statements and (2) the documentation (and later integration into the software) of the different types of referentials used in a class. The first and second avenues are explained in Section 2 and Section 3, respectively.

2 Automated extraction of hypotheses and conclusions from a natural language problem statement

This process is an important addition to the proof generation tool as it will facilitate the task of encoding the problem statement in the QED-Tutrix software. Currently, it is necessary to

*http://turing.scedu.umontreal.ca.

complete the tedious task of writing down each hypothesis including the low-level ones [2], such as "A is a point" or "there is a line named (AB) passing through A and B", which is especially problematic for busy teachers that would like to quickly add a problem in QED-Tutrix. To automate this process, this information will be extracted directly from the problem statement, written in natural language.

Presently, there are few geometric problem solvers that can automatically extract information from problem statements in their natural language environment [6]. As a result, the understanding and extraction of the hypotheses are delegated to the user who must themselves manually formulate them according to the predefined input interface of the problem solver. This manual extraction might give incorrect results due to a wrong or incomplete interpretation by the user. The major challenge of automatic knowledge extraction is the variation in language. Given a geometry problem statement with a set of fixed hypotheses and conclusions, the extraction can be formulated in several ways without modifying or adding new elements. For example, let us consider these two similar problem statements that might be given in a French-speaking class:

1. “Soit ABCD un quadrilatère quelconque, on appelle P, Q, R et S les milieux respectifs des côtés [AB], [BC], [CD] et [DA]. Montre que le quadrilatère PQRS est un parallélogramme.”
   (“Let ABCD be any quadrilateral, we call P, Q, R and S the respective midpoints of the sides [AB], [BC], [CD] and [DA]. Prove that the quadrilateral PQRS is a parallelogram.”)

2. “Dans un quadrilatère ABCD, on relie les milieux P, Q, R et S des segments [AB], [BC], [CD] et [DA]. Montre que le quadrilatère PQRS est un parallélogramme.”
   (“In a quadrilateral ABCD, the P, Q, R and S midpoints are connected to segments [AB], [BC], [CD] and [DA]. Prove that the quadrilateral PQRS is a parallelogram.”)

Each statement contains both the same hypotheses (e.g. "ABCD is a quadrilateral", "P is the midpoint of the line segment [AB]", "Q is the midpoint of the line segment [BC]", "R is the midpoint of the line segment [CD]") and the same conclusion (e.g. "PQRS is a parallelogram"). As depicted in the previous example, other variations in problem statements might be found due to variations in syntax or changes in the order of stated assumptions. Given the potentially very high number of formulations of problem statements, it is important that the automatic extractor should be flexible and have a high tolerance for these linguistic variations.

Another type of variation found in problem statements is the mathematical variation of the hypotheses, where an assumption can be stated in completely different ways while maintaining the same mathematical meaning. For example, the two following assumptions "ABC is a right angle" and "the measure of angle ABC is 90°" are mathematically equivalent, but have been stated in different ways not influenced by linguistic variation. Therefore, the extractor must recognize both these mathematical variations of hypotheses in addition to variations in language. The input states of the problem solvers are finite and limited. Therefore, the extractor must gather equivalent assumptions and place them into equivalence classes, which can be adjusted to these predefined inputs.

3 Documentation of referentials used in class

This task will provide information about which properties are currently used in classrooms. This information will include unusual properties that only a few teachers might use, thereby making the generated proofs feel more natural to the students. In QED-Tutrix, teachers will be capable of dynamically select which properties students can use for a problem at a given time in the school curriculum. The term "referential" is used in the context of the Mathematical Working Space by Kuzniak and Richard [3], where it is the set of properties and definitions used by an individual to solve a problem. We certainly expect this set for a professional mathematician to be bigger than that of an apprentice as it grows as one is learning. The difficulty to document the referentials resides in its dynamic aspect.

In Québec, the ministry is responsible for the curriculum in particular at the high school level (12-17 years old). More specifically, in geometry, the subjects that are to be taught can be summarized by a list of properties. Therefore, there is a first set of sanctioned properties by the ministry, but there is also a second set of suggested properties [4]; thus, we have obligatory and non-obligatory referentials, respectively. This non-obligatory referential is not always used or seen in class, as the referentials in school manuals don't completely match. Although, there is some overlap. At this time, we do not know the exact list used by teachers: is it the one from the ministry, from the school manuals or another personal referential known only by that person? Furthermore, the different properties and definitions are taught in different school years. For example, in Québec, the three cases of similarity of triangles are seen in the fourth year of high school (15-16 years old), the similarity coefficient is seen in the previous third year and the homothety constructions are typically touched in the first or second year [5]. Depending of the school year, students can work with similar concepts, but use different properties.

In these school manuals, we generally find a similar structure in each chapter: exploration activities, class notes, and then exercises. Some also have a referential at the end of the book. As a result, the referential of each chapter precedes the exercises. In some cases, a mathematical problem brings the needs for new properties that are required for its resolution. Similarly, some manuals make the student prove a new property that will be subsequently used in later problems. Therefore, we distinguish two types of referential: (1) the initial referential at the beginning of a chapter and (2) the constructed referential, which contains properties which will be added to a student's referential while they are solving problems. The dynamic nature of the referentials must be considered in the automated solutions of the geometry problems ensuring that QED-Tutrix reflects the reality of what is currently being taught in classrooms.

Keywords
QED-Tutrix, tutoring software, adaptability, automated extraction, referential

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