

Issues and challenges about instrumental proof

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The notion of instrumental proof is relatively new. If the term is little used in didactic literature, its natural association with technologies, old and new, seems self-evident.

On the epistemological side, the discovery of Archimedes's palimpsest recently allowed us to better understand how the weighing method was a kind of mechanical proof, which suggests to the point that the association between proof and artifacts/tools is rather old. Similarly, computer proofs such as those of the four-colour theorem –first shown in 1976 by Kenneth Appel and Wolfgang Haken, then formally addressed in 2005 using Coq software by Georges Gonthier and Benjamin Werner– offer proofs where they are algorithms that base the decision or the verification of all cases, reflecting an unavoidable reality of contemporary mathematical work. Whether they are physical or logical, the use of tools in a validation situation certainly renews the usual idea that we have between the concepts of proof, modelling and representation of knowledge.

On the didactic side, there seems to be a constant struggle with paradoxes. The student is asked to prove propositions, but he or she now has an automated reasoning tool. It requires him or her to work with meaningful knowledge and to transform it, but by working more and more at the interface of computer tools that manage both a part of the representation and treatment, and often even experimenting on mathematical objects (e.g. dynamic figures) as a physicist does with objects of his own domain. And all this, without the teacher can refer to mathematics that could be described as technological, since he was initiated to a deductive science that has developed traditionally in writing.

It is then by extending ideas that we have already exposed in our work, including the recent paper [3], *The Concept of Proof in the Light of Mathematical Work*, and resuming conclusions of our current research projects (design of the tutorial system QED-Tutrix in high school geometry [1], the use of the Automated Reasoning Tools (ART) [2] in teacher training) that we approach the question of instrumental proofs. With this attitude, the subject-milieu interaction is a unit of epistemic necessity, the subject can be both a reader, to consider traditional proofs, and the user of software or a mathematical machine. The notions of reasoning in action and reasoning that unfold differently than with the discourse will be treated, as well as the theory of mathematical working spaces in which the question of the coordination of discursive, semiotic and instrumental geneses arise between an epistemological and a cognitive plan.

Keywords: Instrumental proof, mathematical working space, instrumented reasoning, algorithmic, physics

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