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# Discovering properties of bar linkage mechanisms based on partial Latin squares by means of Dynamic Geometry Systems

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Dynamic Geometry Systems (DGSs) have recently been proposed in mechanical engineering as an alternative to deal with the teach, design, analysis and implementation of mechanisms [1, 6, 7, 8, 9]. Recall that a *mechanism* is any set of rigid bodies connected by *joints* so that force and motion are transmitted among themselves. A *link* within a mechanism is any of its rigid bodies having at least two different joints. A *bar linkage mechanism* is any mechanism in which all its rigid bodies are bars and at least one of them is a link. The study of the relative motion that occurs between each pair of connected bars within one such a mechanism enables its characterization. In this regard, the *degree of freedom of a joint* connecting two bars is defined as the number of independent parameters that are required to determine the relative position of one of the bars with respect to the other one. This has influence on the different *coupler curves* that are generated by the joints within each bar. The study and analysis of such curves enable one to design optimal devices and give rise, therefore, to important applications in Technology and Engineering. Since coupler curves can be described as loci of points satisfying certain geometrical constraints derived from the lengths and connections of bars within a mechanism, DGSs constitute an interesting tool to investigate and characterize them from a visual and dynamical point of view.

In this work, we focus on the use of DGSs to deal with those bar linkage mechanisms such that the distance matrix defined by their joints constitutes a unipotent partial Latin square satisfying certain conditions. Recall that a *partial Latin square* of order $n$ is an $n \times n$ array in which each cell is either empty or contains an element of a finite set of $n$ symbols so that each symbol occurs at most once in each row and in each column. Let $\mathrm{PLS}(n)$ denote the set of partial Latin squares of order $n$ having $[n] := \{0, 1, \ldots, n-1\}$ as set of symbols. The rows and columns of every array in $\mathrm{PLS}(n)$ are supposed to be naturally indexed by the elements of the set $[n]$. Throughout our study, we focus on the subset of partial Latin squares $L = (l_{ij}) \in \mathrm{PLS}(n)$ that are also

i. *reduced*, that is, such that $l_{0i} = i$ and $l_{j0} = j$, for all $i, j \in [n]$ satisfying that the cells $(0, i)$ and $(j, 0)$ in $L$ are non-empty;

ii. *zero-diagonal*, that is, $l_{ii} = 0$, for all $i \in [n]$; and

iii. *symmetric*, that is, $l_{ij} = l_{ji}$, for all $i, j \in [n]$.

To avoid degeneracy and disjoint unions of disconnected mechanisms, we also suppose that

iv. every row and every column of $L$ must contain at least one symbol of the set $[n] \setminus \{0\}$;

v. for each pair $(i, j) \in [n] \times [n]$ such that $l_{ij} \in [n]$, there exists a positive integer $k \in [n]$ such that either $l_{kj} \in [n]$ or $l_{ik} \in [n]$. This involves every bar in the mechanism to be connected to at least one other bar by a joint.

Finally, in order to get linkage mechanisms, the following condition is also required:

vi. If every symbol in $[n] \setminus \{0\}$ appears exactly twice in $L$, then they cannot be all of them in a same row and column of $L$.

Let $\mathcal{M}_n$ denote the set of partial Latin squares of order $n$ satisfying Conditions (i)–(vi). This set is preserved by isomorphisms. Recall that two partial Latin squares $L = (l_{ij})$ and $L' = (l'_{ij})$ in $\mathrm{PLS}(n)$ are *isomorphic* if there exists a permutation $\pi$ on the set $[n]$ such that $\pi(l_{ij}) = l'_{\pi(i)\pi(j)}$, for all $i, j \in [n]$ such that $l_{ij} \in [n]$. To be isomorphic constitutes an equivalence relation among partial Latin squares. The distribution of partial Latin squares into isomorphism classes is known [2, 3, 5], for order $n \leq 6$.

Let $M(L)$ denote the set of bar linkage mechanisms that are associated to a given partial Latin square $L = (l_{ij}) \in \mathcal{M}_n$ as follows:

1. There exists a bar $B_{ij}$ within the mechanism, for each pair $(i, j) \in [n] \times [n]$ such that $i < j$ and $l_{ij} \in [n] \setminus \{0\}$.

2. Two different bars $B_{ij}$ and $B_{i'j'}$ within the mechanism are connected by a joint $J_k$ if and only $\{i, j\} \cap \{i', j'\} = \{k\} \neq \emptyset$. This joint is placed in the corresponding extreme of each bar.

3. Two different bars $B_{ij}$ and $B_{i'j'}$ within the mechanism have the same length if and only if $l_{ij} = l_{i'j'}$.

DGSs constitutes an interesting tool to deal with the study, analysis and characterization of the bar linkage mechanisms in the set $M(L)$. To this end, we consider each symbol $k \in [n] \setminus \{0\}$ to be uniquely associated to a slider $s_k$ so that the length of each bar $B_{ij}$ such that $l_{ij} = k$ is the value given by such a slider $s_k$ (see Figure 1).

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | | 3 |
| 1 | 0 | | 2 | |
| 2 | | 0 | 4 | |
| | 2 | 4 | 0 | |
| 3 | | | | 0 |

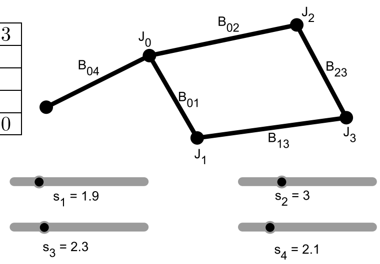


Figure 1: Dynamical study of a bar linkage mechanism based on a partial Latin square.

In this work, we make use of the mentioned sliders to teach, investigate properties and formulate conjectures about lengths of bars and coupler curves related to those mechanisms associated to partial Latin squares in the set $\mathcal{M}_n$, according to their distribution into isomorphism classes. In this regard, remark the recent study [4] about loci of points whose distance matrix constitutes a partial Latin square satisfying Conditions (i)–(iii). Further, the inclusion on new sliders within each worksheet under consideration enables us to deal with different parameters that characterize our bar linkage mechanisms, as the degree of freedom, the transmission ratio, or the mechanical advantage, amongst others. All the constructions that have been developed in this work are available online in the official repository of GEOGEBRA, at the address `https://www.geogebra.org/m/crvJ7CzX`.

**Keywords:** Linkage systems, dynamic geometry, partial Latin square.

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