Hoja nº_____

1. $f(x) = \int_1^x e^{-t^2}\,dt \qquad x > 0$

① $f'(x) = e^{-x^2}$

$f^{2)}(x) = -2x\, e^{-x^2}$

$f^{3)}(x) = -\left(2e^{-x^2} + (2x)(-2x)e^{-x^2}\right) = -(2-4x^2)e^{-x^2} = -2(1-2x^2)e^{-x^2}$

$T_3(f(x),1)(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f^{2)}(1)}{2!}(x-1)^2 + \frac{f^{3)}(1)}{3!}(x-1)^3 =$

$= 0 + \frac{1}{e}(x-1) - \frac{\cancel{2}}{e\cancel{2!}}(x-1)^2 + \frac{2}{e3!}(x-1)^3$

② $\left| f(x) - \int_1^x e^{-t^2} \right| = \left| \frac{f^{3+1)}(\xi_x)}{4!}(x-1)^4 \right| = \left| \frac{-4\xi_x e^{-\xi_x^2}(2\xi_x^2-3)}{4!}(x-1)^4 \right| = (*)$

$f^{4)}(x) = -2\left[-4x\,e^{-x^2} + (1-2x^2)(-2x)e^{-x^2}\right] = -2e^{-x^2}\left[(1-2x^2)(-2x) - 4x\right] =$

$= -2e^{-x^2}\left[-2x + 4x^3 - 4x\right] = -4e^{-x^2}(2x^3 - 3x) = -4x\,e^{-x^2}(2x^2-3)$

$(*)\quad \frac{4\xi_x e^{-\xi_x^2}}{4!}\,|2\xi_x - 3|\,|(x-1)^4| \leq \frac{4 \cdot 2e^{-1}}{4!}\,|2\xi_x - 3| = \frac{\cancel{4}\cdot\cancel{2}\cdot 1/e}{\cancel{4}\cdot 3\cdot\cancel{2}} = \boxed{\frac{1}{3e}}$ ✓

e^{-x^2} decreciente y >0

ξ_x entre 1 y x $\Rightarrow$ $e^{-4} < e^{-\xi_x^2} < e^{-1}$

$x \in [1,2]$

$1 \leq \xi_x \leq 2$

2.-

$$f(x,y) = \frac{\cos x \ \operatorname{sen} y}{\sqrt{x}}$$

$$\frac{\partial f}{\partial x} = \frac{-(\operatorname{sen} x \ \operatorname{sen} y)\sqrt{x} - (\cos x \ \operatorname{sen} y)\frac{1}{2}x^{-\frac{1}{2}}}{x}$$

$$\frac{\partial f}{\partial y} = \frac{\cos x \cos y}{\sqrt{x}}$$

$$\Delta f = \left| \frac{-\left[(\operatorname{sen} \tilde{\pi} \ \operatorname{sen} \tilde{\sqrt{2}})\sqrt{\tilde{\pi}} + (\cos \tilde{\pi} \ \operatorname{sen} \tilde{\sqrt{2}})\frac{1}{2}\sqrt{\tilde{\pi}}\right]}{\tilde{\pi}} \right| |\tilde{\pi} - \pi| +$$

$$+ \left| \frac{\cos \tilde{\pi} \cos \tilde{\sqrt{2}}}{\sqrt{\tilde{\pi}}} \right| |\tilde{\sqrt{2}} - \sqrt{2}| \quad < \quad 0{,}5 \times 10^{-6} \quad \Rightarrow$$

$$\Delta f < \frac{\sqrt{\tilde{\pi}} + \frac{1}{2}\sqrt{\tilde{\pi}}}{\tilde{\pi}} |\tilde{\pi} - \pi| + \frac{1}{\sqrt{\tilde{\pi}}} |\tilde{\sqrt{2}} - \sqrt{2}| < |\tilde{\pi} - \pi| + |\tilde{\sqrt{2}} - \sqrt{2}| <$$

$$< \quad 0{,}5 \times 10^{-5}$$

si $\tilde{\pi}$ y $\tilde{\sqrt{2}}$ son aproximacions a 6 cifras decimals exactas lo tenemos.

Hoja nº_____

(3) $f_1(x) = x^2 - \frac{1}{2}$, $x > 0$

$f_2(x) = \log(1+x)$, $x > 0$

a) $x^2 - \frac{1}{2} = \log(1+x)$

$f(x) = x^2 - \log(1+x) - \frac{1}{2}$

$f(0) = -\frac{1}{2} < 0$

$$\left\{ \begin{array}{l} f(1) = 1 - \log 2 - \frac{1}{2} = 1 - 0,6932 - 0,5 < 0 \\ f(1.5) = 1.5 - \log 2.5 - 0,5 = 1.5 - 0,9163 - 0,5 > 0 \end{array} \right] \circledast$$

$f'(x) = 2x - \frac{1}{1+x} = \frac{2x + 2x^2 - 1}{1+x} = 0 \iff$

$x = \frac{-1 \pm \sqrt{3}}{2}$

En $(0, +\infty)$ $f'(x) = 0 \iff x = \frac{-1+\sqrt{3}}{2} \sim 0,3661$

$(0, \frac{-1+\sqrt{3}}{2})$ $f' < 0 \Rightarrow f$ no tiene raíces

$(\frac{-1+\sqrt{3}}{2}, +\infty)$ $f' > 0 \Rightarrow f$ es inyectiva $+$ Ⓐ $\Rightarrow$

f tiene una única raíz > 0 que está en $[1, 1.5]$.

b) $|\xi_n - \xi| < \frac{1}{2^n} |1.5 - 1| = \frac{0.5}{2^n} < 0.5 \times 10^{-3} \iff \frac{1}{2^n} < 10^{-3}$

$n \geq 10$

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c)

$$f(x) = x^2 - \log(x+1) - \frac{1}{2} = 0 \iff x^2 = \log x + \frac{1}{2} \iff$$

$$x = \pm\sqrt{\log(x+1) + 1/2}$$

$$g(x) = +\sqrt{\log(x+1) + 1/2}$$

P.1)

$$g(1) = \sqrt{\log 2 + 1/2} \approx 1.090 \in [1, 1.5]$$

$$g(1.5) = \sqrt{\log 2.5 + 1/2} \approx 1.1901 \in [1, 1.5]$$

$$g'(x) = \frac{1}{2}\left(\log(x+1) + 1/2\right)^{-1/2} \cdot \frac{1}{1+x} =$$

$$= \frac{1}{2\sqrt{\log(1+x) + 1/2}\,(1+x)} > 0 \quad x \in [1, 1.5] \quad \Rightarrow$$

$$\Rightarrow \quad 1 \le g(x) \le 1.5$$

$$1 \le x \le 1.5$$

P.2)

$$\left| \frac{1}{2\left(\log(1+x) + 1/2\right)^{1/2}(1+x)} \right| \le \frac{1}{2^2\left(\log 2 + \frac{1}{2}\right)^{1/2}} \approx \frac{1}{2^2 \cdot (1{,}1932)^{1/2}} \approx$$

$$\approx < 0{,}23 = k < 1$$

(*) $$\boxed{\left(\log 2 + \frac{1}{2}\right)^{1/2} \le \underbrace{\left(\log(1+x) + \frac{1}{2}\right)^{1/2}}} \le \left(\log(2.5) + 1/2\right)^{1/2}$$

Función creciente. $y > 0$ cuando $x \in [1, 1.5]$

$$\frac{1}{\left(\log(1+x) + 1/2\right)^{1/2}} \le \frac{1}{\left(\log 2 + 1/2\right)^{1/2}}$$

(**)

$$\boxed{\begin{gathered} 2 \le x+1 \le 2.5 \\ \Downarrow \\ \frac{1}{2.5} \le \frac{1}{x+1} \le \frac{1}{2} \end{gathered}}$$

(P.1) + (P.2) nos dicen que hay convergencia para $g(x) = +\sqrt{\log(x+1)+1/2}$ en todos los puntos de $[1, 1.5]$.

Hoja nº

d)

$$|\xi_n - \xi| \leq \frac{k^n}{1-k}|1-1.5| = \frac{(0,23)^n}{0.77} \times 0.5 < 0.5 \times 10^{-4}$$

$n \geq 7$.

$\xi_0 = 1$

$\xi_1 = \sqrt{\log 2 + 1/2} \sim 1.0923$

$\xi_2 = \sqrt{\log(2.0923)+1/2} \sim 1,1128$

$\xi_3 = \sqrt{\log(2,1128)+1/2} \sim 1,1171$

$\xi_4 = \sqrt{\log(2,1171)+1/2} \sim 1,1181$

$\boxed{\xi_5 = \sqrt{\log(2,1181)+1/2} \sim 1,1183}$ ✓