Discovering properties of bar linkage mechanisms based on partial Latin squares by means of DGSs

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ACA 2018 (Santiago de Compostela. June 20, 2018).

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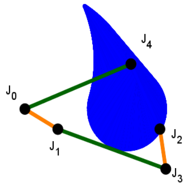
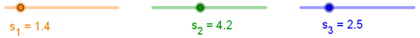
1	0		2	
		0	1	3
	2	1	0	
2		3		0





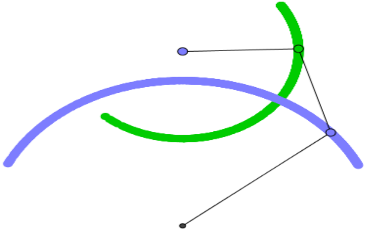
CONTENTS

I. Linkages derived from PLS

Mechanisms.

- **Mechanism**: Set of rigid bodies connected by *joints* and transmitting force and motion.

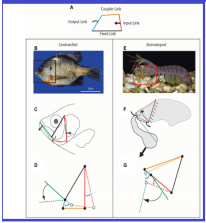
- **Link**: Rigid body having two joints.
- **Bar linkage mechanism**: Rigid bodies = Bars. At least one link.
- **Coupler curve**: Trace curve generated by a joint.

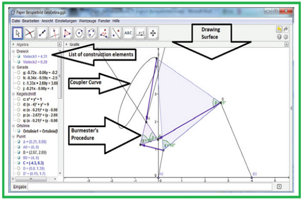


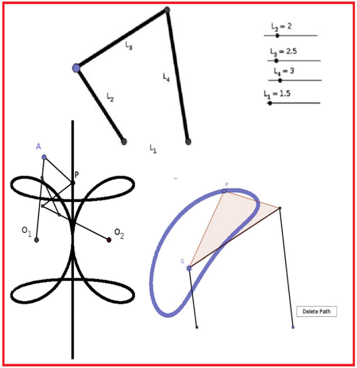
Synthesis and analysis of mechanisms by using DGSs.

Common evolutionary trends underlie the four-bar linkage systems of sunfish and mantis shrimp

Yinan Hu,[1] Nathan Nelson-Maney,[2] and Philip S. L. Anderson[3,4] 2017







Comparison of Geometry Software for the Analysis in Mechanism Theory

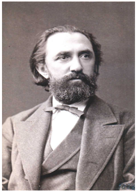
S. Kurtenbach, I. Prause, C. Weigel and B. Corves 2014

Teaching Mechanism and Machine Theory with GeoGebra

X. Iriarte, J. Aginaga and J. Ros 2014

Kinematics

- **Kinematics**: **Description of the motion** of a mechanism without considering neither its cause nor the mass of its components.
  For each point: **Position**, *velocity* and *acceleration*.



Franz Reuleaux.

(Germany, 1829–1905).

1876:

Kinematic chain: Mechanism.

Kinematic pair: Joint.

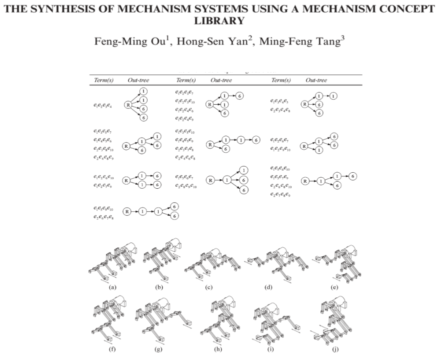
Every constraint on a kinematic chain can be described as a system of constraints on its kinematic pairs.

Classification parameters of mechanisms:

- **Degree-of-freedom**: Minimum number of parameters defining its configuration (coordinates and motion).
- Number of links.
- Number of joints.
- Types of joints: screw, wheel, cam, crank, belt and ratchet.

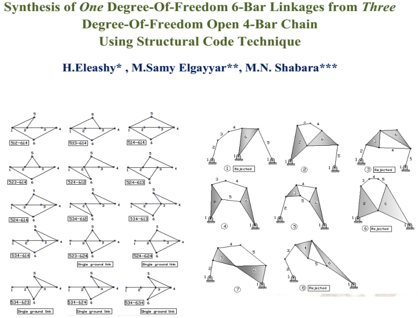
- **Kinematic diagram**: **Graphical representation** of a mechanism, which illustrates the connectivity of links and joints.
- **Linkage graph**: Graph $G = (V, E)$ such that:
  - $V \equiv$ Joints.
  - $E \equiv$ Links.

Graphical enumeration technique


THE SYNTHESIS OF MECHANISM SYSTEMS USING A MECHANISM CONCEPT LIBRARY

Feng-Ming Ou[1], Hong-Sen Yan[2], Ming-Feng Tang[3]

2012


Synthesis of *One* Degree-Of-Freedom 6-Bar Linkages from *Three* Degree-Of-Freedom Open 4-Bar Chain Using Structural Code Technique

H.Eleashy* , M.Samy Elgayyar**, M.N. Shabara***

2010

Partial Latin squares (PLS(n)).

- **Partial Latin square**: $n \times n$ array whose cells are empty or contains an element of $[n] := \{0, \ldots, n-1\}$, without repetitions per row or column.

$$L = (l_{ij}) \equiv \begin{array}{|c|c|c|c|c|}\hline & 1 & 3 & & 4 \\ \hline 1 & 0 & & 2 & \\ \hline 2 & & 1 & 3 & \\ \hline 4 & 2 & 0 & & 1 \\ \hline & 3 & 2 & & \\ \hline \end{array}$$

n	# PLS(n)	# LS(n)
1	2	1
2	35	2
3	11776	12
4	127545137	576
5	64170718937006	161280
6	2027032853070203981647	812851200
7	5175166233060627523665748739420	61479419904000
8	$*$	108776032459082956800
9	$*$	5524751496156892842531225600
10	$*$	9982437658213039871725064756920320000
11	$*$	776966836171770144107444346734230682311065600000
	[F13, F15, FS18]	

- $L = (l_{ij})$ and $L' = (l'_{ij})$ in $\mathrm{PLS}(n)$ are **isomorphic** if $\exists \pi \in S_n$ such that

$$\pi(l_{ij}) = l'_{\pi(i)\pi(j)}, \forall i, j \in [n] \text{ such that } l_{ij} \in [n].$$

0	1	
1	0	2
	2	0

0	1	2
1	0	
2		0

n	# Isomorphism classes $\mathrm{PLS}(n)$
1	2
2	20
3	2029
4	5319934
5	534759300183
6	281532343587241 0905

[F13, F15, FS18]

A subset of partial Latin squares ($\mathcal{M}_n$).

$\boxed{\mathcal{M}_\mathbf{n}}$

- **Reduced**: $l_{0i}, l_{i0} \in \{\emptyset, i\}$, for all $i \in [n]$.
- **Zero-diagonal**: $l_{ii} = 0$, for all $i \in [n]$
- **Symmetric**: $l_{ij} = l_{ji}$, for all $i, j \in [n]$.
- There exists at least one non-zero symbol per row and per column.
- Each non-zero symbol of $[n]$ appears at least twice.
- $l_{ij} \in [n] \setminus \{0\} \Rightarrow \exists k \in [n]$ such that $\{l_{kj}, l_{ik}\} \cap ([n] \setminus \{0\}) \neq \emptyset$.
- If every non-zero symbol appears exactly twice, not all of them are in the same row or column.

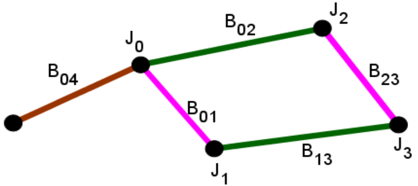
$L = (l_{ij}) \equiv$

0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0

M(L): Set of bar linkage mechanisms derived from $L = (l_{ij}) \in \mathcal{M}_n$ as follows:

- There exists a bar B_{ij} if $l_{ij} \in [n] \setminus \{0\}$ $(i < j)$.
- B_{ij} and B_{ik} are connected by a joint J_i.
- B_{ij} and B_{kj} are connected by a joint J_j.
- If $l_{ij} = l_{i'j'}$, then $|B_{ij}| = |B_{i'j'}|$.

0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



The **distance matrix** related to the joints is derived from L and $\{|B_{ij}|\}$.

0	$\lvert B_{12}\rvert$	$\lvert B_{13}\rvert$	0	$\lvert B_{15}\rvert$
$\lvert B_{12}\rvert$	0	0	$\lvert B_{13}\rvert$	0
$\lvert B_{13}\rvert$	0	0	$\lvert B_{15}\rvert$	0
0	$\lvert B_{13}\rvert$	$\lvert B_{15}\rvert$	0	0
$\lvert B_{15}\rvert$	0	0	0	0

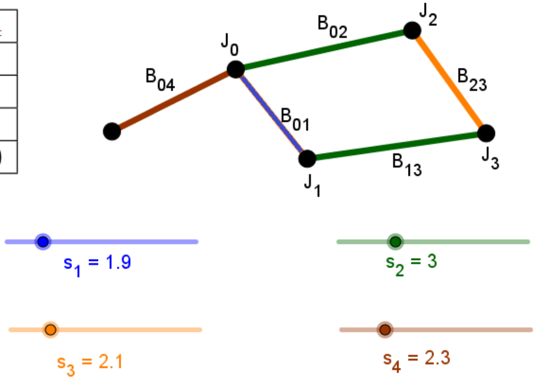
II. Analysis by DGS

Representation in a DGS of bar linkages based on a PLS

$L = (l_{ij}) \in \mathcal{M}_n$.

- Each symbol $k \in [n] \setminus \{0\}$ is uniquely associated to a slider s_k.

0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



Representation in a DGS of bar linkages based on a PLS

https://www.geogebra.org/m/crvJ7CzX

$$|\mathcal{M}_4| = 7 \qquad |\mathcal{M}_5| = 43$$

GeoGebra

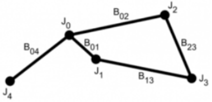
Bar linkage mechanisms based on partial Latin squares

Autor: Raúl Manuel Falcón Ganfornina

This GeoGebra Book contains different worksheets related to the study, analysis and characterization of bar linkage mechanisms associated to a given reduced, zero-diagonal and symmetric partial Latin square. The GeoGebra Book is distributed into chapters according to the order of the partial Latin square under consideration.

Reference:

R. M. Falcón, Discovering properties of bar linkage mechanisms based on partial Latin squares by means of Dynamic Geometry Systems. In: 24th Conference on Applications of Computer Algebra ACA 2018. (Santiago de Compostela, June 18-22, 2018).



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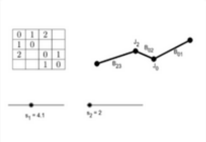
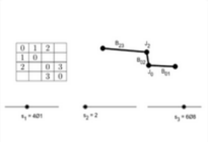
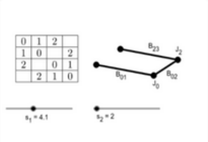
Representation in a DGS of bar linkages based on a PLS



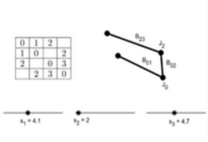
M(L_{4,1})



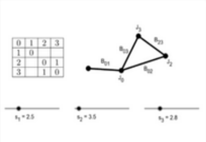
M(L_{4,2})



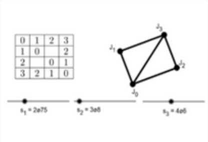
M(L_{4,3})



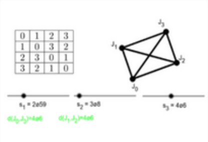
M(L_{4,4})



M(L_{4,5})



M(L_{4,6})

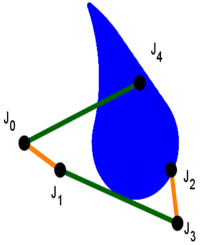


M(L_{4,7})

Representation in a DGS of bar linkages based on a PLS

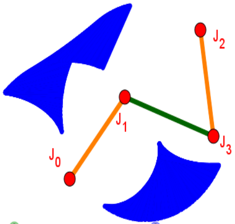
Analysis of a bar linkage ($M_{5,1}$).

0	1			2
1	0		2	
		0	1	3
	2	1	0	
2		3		0



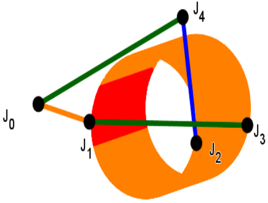
$s_1 = 1.4$ $s_2 = 4.2$ $s_3 = 2.5$

0	1			2
1	0		2	
		0	1	3
	2	1	0	
2		3		0



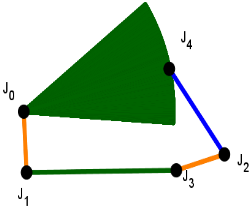
$s_1 = 3.8$ $s_2 = 4.2$ $s_3 = 2.5$

0	1			2
1	0		2	
		0	1	3
	2	1	0	
2		3		0



$s_1 = 1.4$ $s_2 = 4.2$ $s_3 = 2.5$

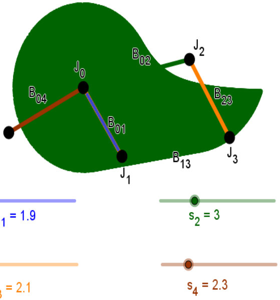
0	1			2
1	0		2	
		0	1	3
	2	1	0	
2		3		0



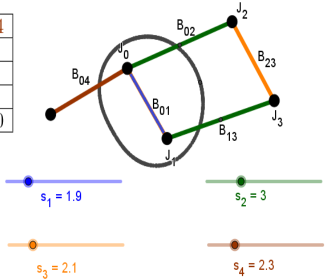
$s_1 = 1.4$ $s_2 = 4.2$ $s_3 = 2.5$

Analysis of a bar linkage ($M_{5,10}$).

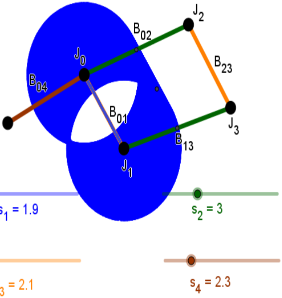
0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



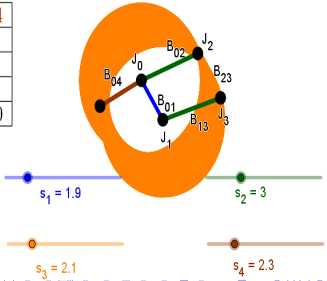
0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



0	1	2		4
1	0		2	
2		0	3	
	2	3	0	
4				0



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F18 R. M. Falcón, Two-dimensional loci of points with a partial Latin square within their distance matrix. Submitted, 2018.

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H17 Y. Hu, N. Nelson-Maney, P. S. L. Anderson, Common evolutionary trends underlie the four-bar linkage systems of sunfish and mantis shrimp. *Evolution* **71**(5), 1397–1405 (2017).

I14 X. Iriarte, J. Aginaga, J. Ros, Teaching Mechanism and Machine Theory with GeoGebra. In *New Trends in Educational Activity in the Field of Mechanism and Machine Theory*, J. C. García-Prada, C. Castejón C. (eds.), 211–219, Springer International Publishing, Switzerland, 2014.

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P15 I. Prause, J. C. Fauroux, M. Hüsing, B. Corves, Using Geometry Sketchers and CAD Tools for Mechanism Synthesis. In *Proceedings of IFToMM 2015, The 14th World Congress in Mechanism and Machine Science*, paper OS3-032, 11 pp., International Federation for the Theory of Mechanisms and Machines, Taiwan, 2015.

R76 F. Reuleaux, *Kinematics of machinery, outlines of a theory of machines*, McMillan and Company, London, 1876.

Many thanks!

Discovering properties of bar linkage mechanisms based on partial Latin squares by means of DGSs

0	1			2
1	0		2	
		0	1	3
	2	1	0	
2		3		0