The Roland and Astrid Dana-Picard Chair for Mathematics, Education and Judaism

JERUSALEM COLLEGE OF TECHNOLOGY
LEV ACADEMIC CENTER

ACA 2018 Santiago de Compostela

PRIVATE PÄDAGOGISCHE HOCHSCHULE DER DIÖZESE LINZ

A new approach to automated study of isoptic curves

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Special session on

Dynamic Geometry and Mathematics Education

Santiago de Compostela

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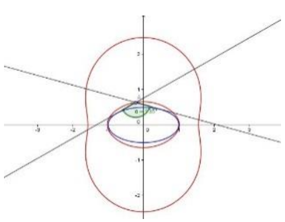
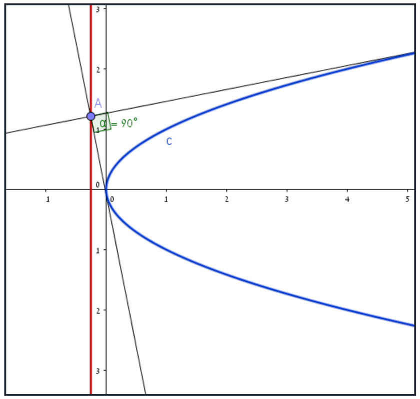
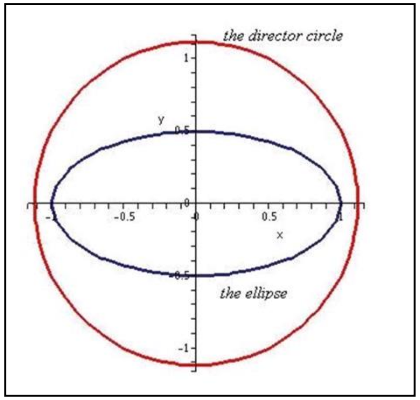
Definition

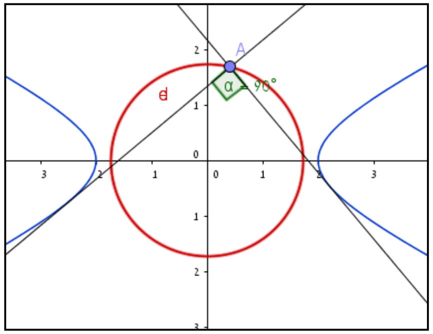
- Let *C* be a plane curve. For a given angle θ (with 0 ≤ θ ≤ 180°), a θ-isoptic of *C* is the geometric locus of points in the plane through which pass a pair of tangents with an angle of θ between them.
- The special case for which θ = 90° is called an orthoptic curve.



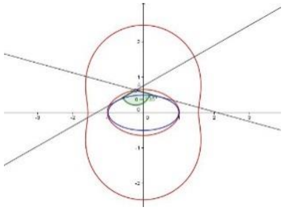
Orthoptics of conics







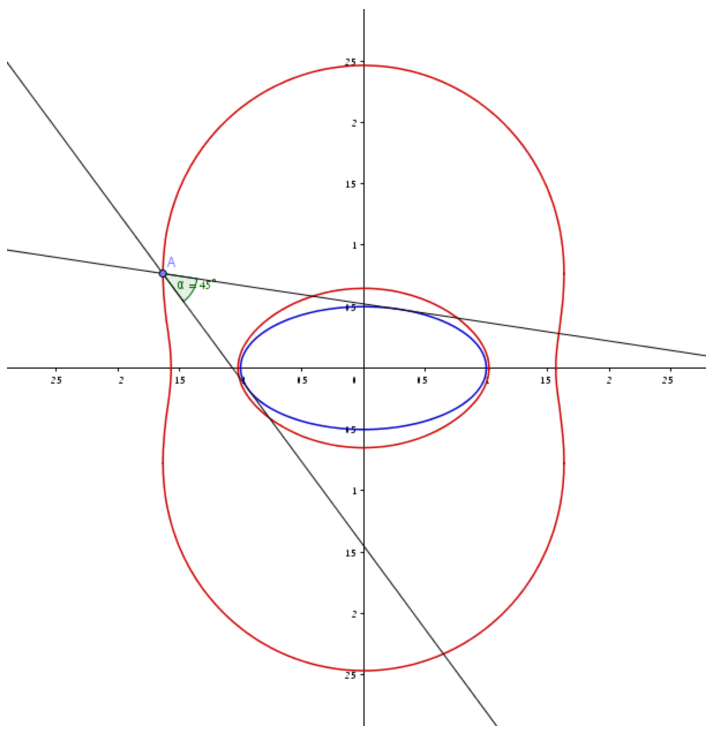
- The directrix of a parabola (always exists).
- The director circle of an ellipse (always exists).
- The director circle of a hyperbola (exists under a condition on the angle between the asymptotes).

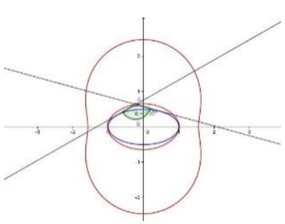


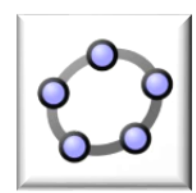
Bisoptics of ellipses

$$E: x^2 + 4y^2 = 1$$

$$Opt(E, 45-135): (x^2 + y^2)^2 - \frac{7}{2}x^2 - \frac{13}{2}y^2 + \frac{41}{16} = 0$$

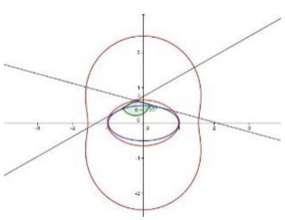






Jordan curves

- A plane curve C which is smooth, strictly convex and closed is called a **Jordan curve**.
- **Theorem:** A Jordan curve divides the plane into three regions, namely the interior, the curve itself and the exterior.
- If the Jordan curve C is strictly convex, then through an interior point, no tangent to C passes, and through an exterior point passes one pair of tangents.



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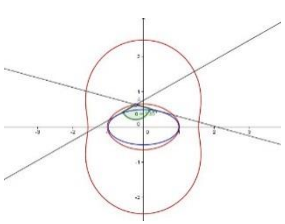
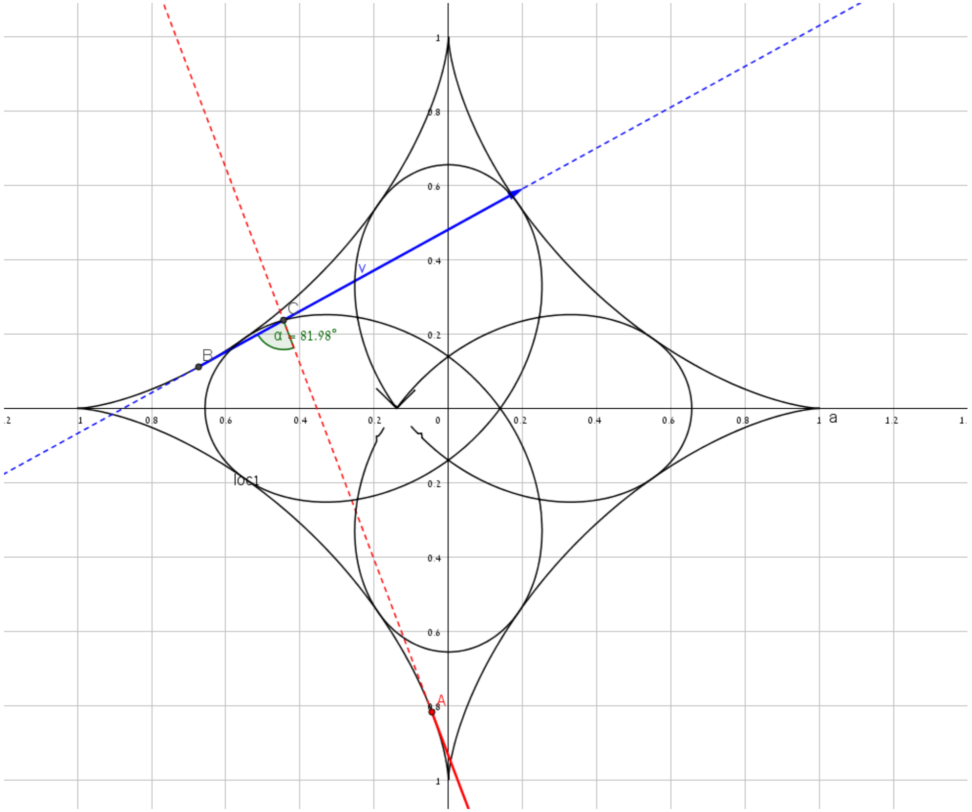
What happens for:

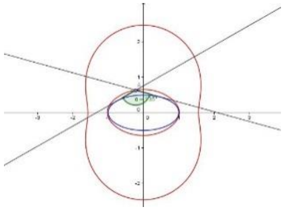
Non closed curves?

Non smooth curves?

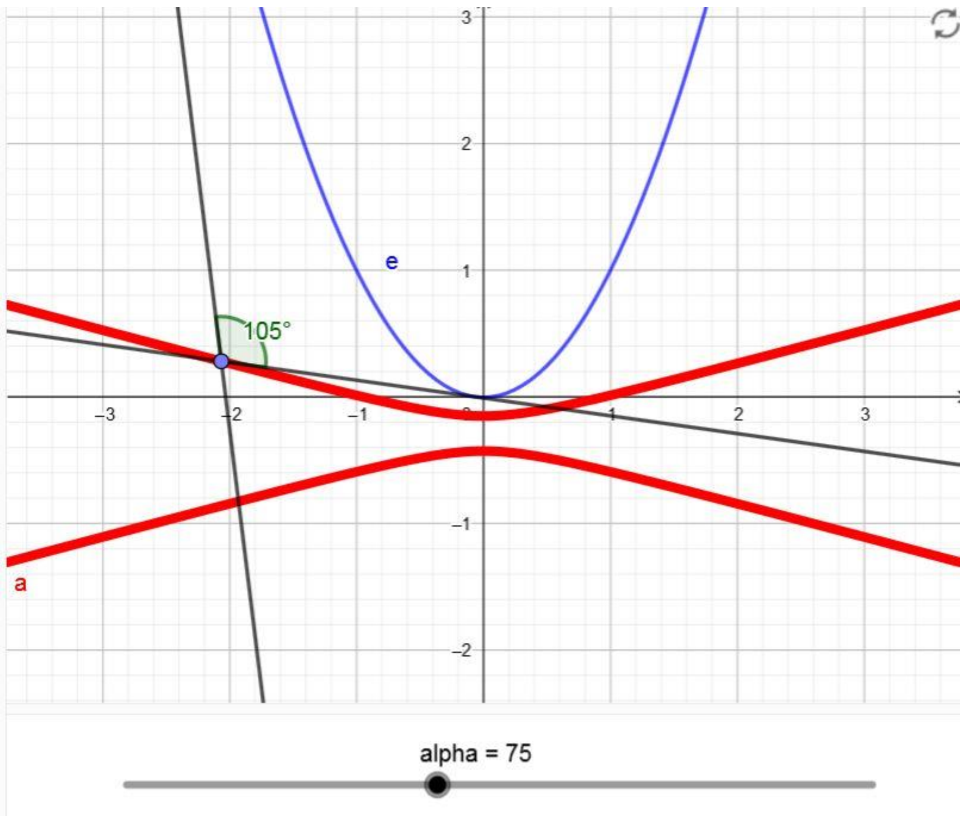


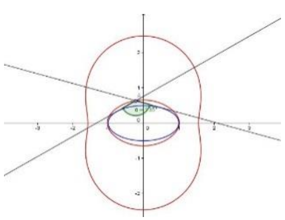
Example 1: Isoptics of an astroid
parametric presentations





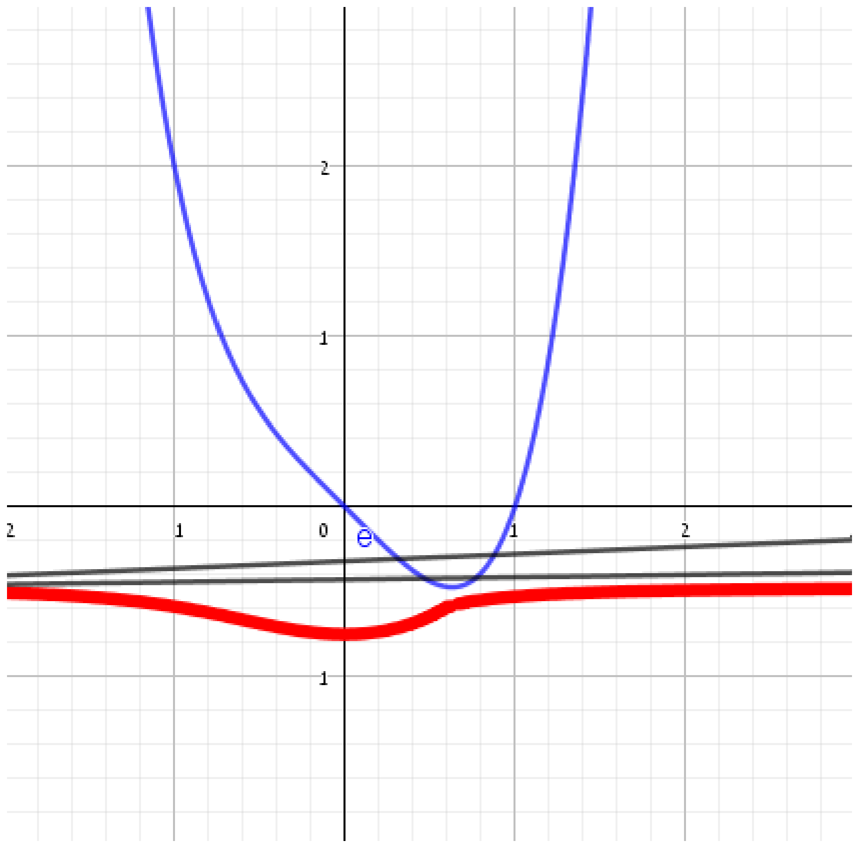
Example 2
105°-isoptic of a parabola

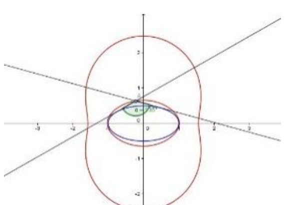




Orthoptic of an open quartic

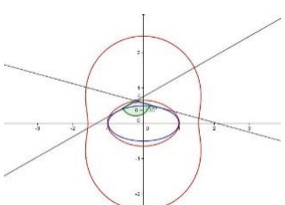
$$y = x^4 - x$$





Two approaches

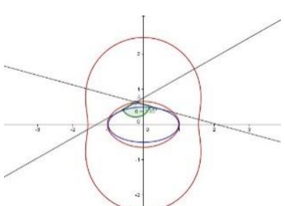
- Parametric method
  - Define the input curve with a parametric presentation
  - Find a presentation for tangents vectors/lines
  - Find an expression for orthogonality of two tangents
  - Compute a parametric presentation of the isoptic
  - Compute an implicit equation by elimination
- Implicit method
  - Define the input curve as an algebraic equation
  - Compute partial derivatives at two hypothetical tangent points
  - Assume that the angle between the tangents is as required
  - Compute an implicit equation by elimination



Two approaches (comparison)

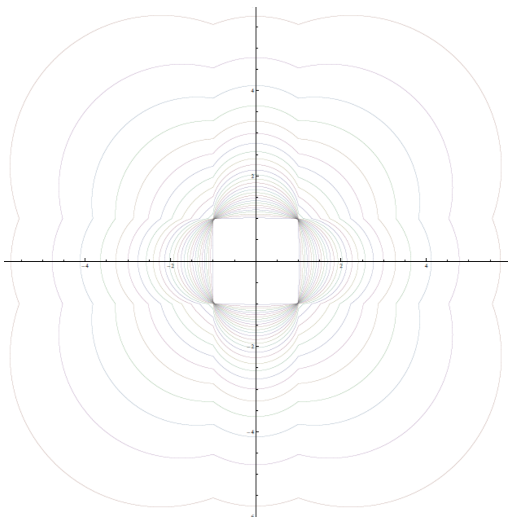
- Parametric method
  - Exact
  - Fast
  - Works only in some special cases

- Implicit method
  - Works in all cases when the degree is low
  - Computationally heavy from quartic cases (Gröbner bases)



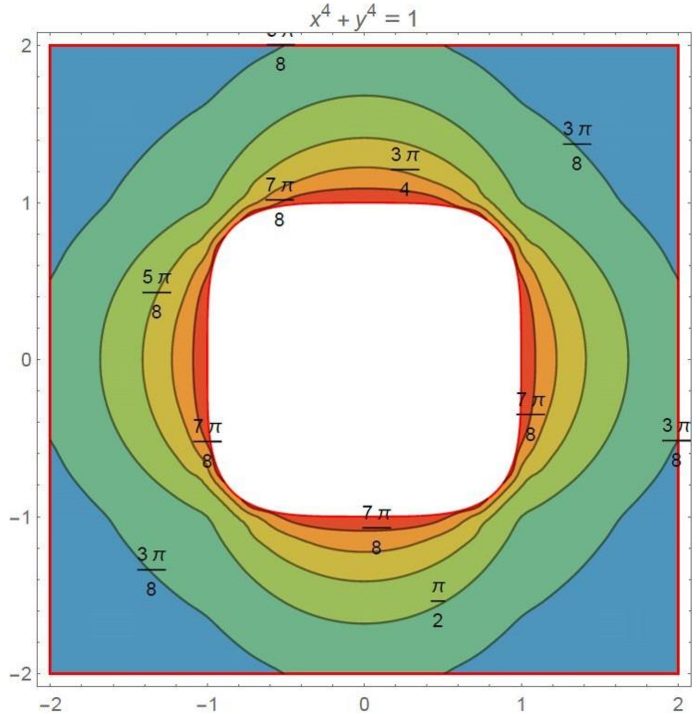
Using Locus and LocusEquation commands

- Example: orthoptic of a closed Fermat curve

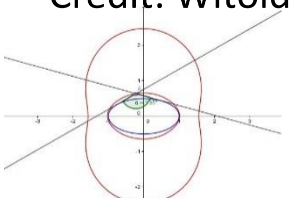


$$x^{40} + y^{40} = 1$$

With Mathematica.
Credit: Witold Mozgawa, Lublin



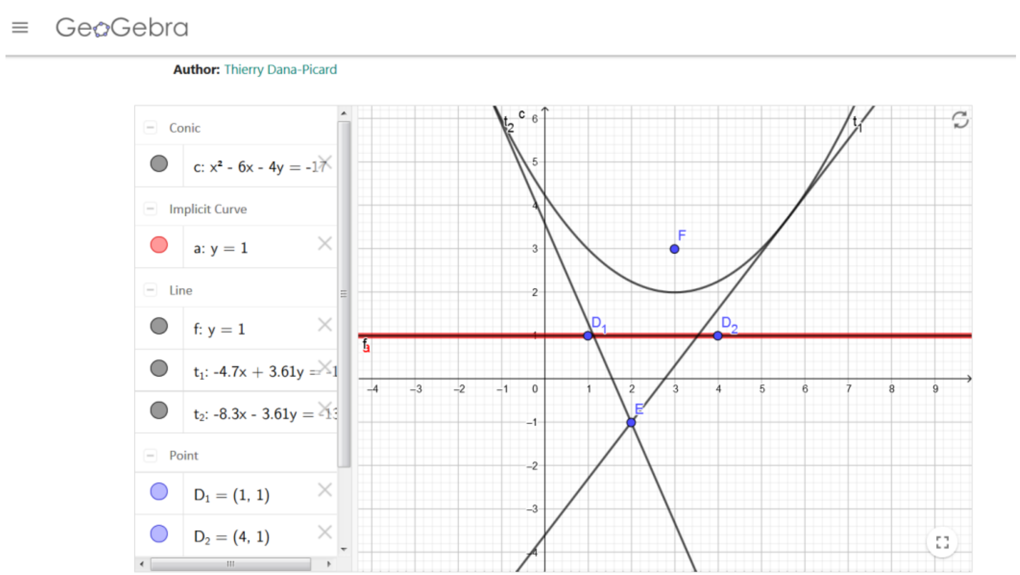
Numerical methods (DP-N)



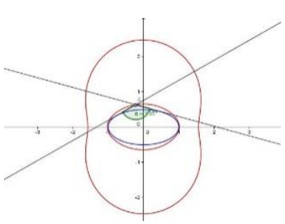
Floor, entrance to an old synagogue, Budapest

Orthoptic of a quartic
using LocusEquation

- Please see https://www.geogebra.org/m/J7tNfrMX



Joint work with Zoltan Kovacs



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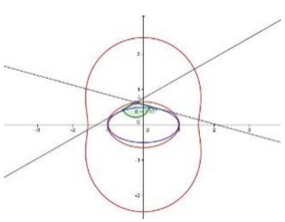
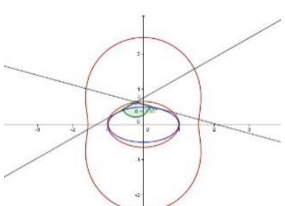


Thank you for your vision and audition!



Thank you for your vision and audition!

And, of course, for your attention

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